Preprint typeset in JHEP style. - HYPER VERSION

SISSA-16/2007/EP

Stringy Instantons at Orbifold Singularities

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Abstract: We study the effects produced by D-brane instantons on the holomorphic quantities of a D-brane gauge theory at an orbifold singularity. These effects are not limited to reproducing the well known contributions of the gauge theory instantons but also generate extra terms in the superpotential or the prepotential. On these brane instantons there are some neutral fermionic zero-modes in addition to the ones expected from broken supertranslations. They are crucial in correctly reproducing effects which are dual to gauge theory instantons, but they may make some other interesting contributions vanish. We analyze how orientifold projections can remove these zero-modes and thus allow for new superpotential terms. These terms contribute to the dynamics of the effective gauge theory, for instance in the stabilization of runaway directions.

Keywords: Instantons, D-branes.

Contents

1. Introduction

It has long been realized that instantons in string theory are often in close correspondence with instantons in gauge theories [1, 2, 3, 4, 5, 6]. Recently it was found that in some situations stringy instantons can dynamically generate some terms which from a low-energy effective point of view enter as ordinary external couplings in the superpotential of gauge theories living on space-filling branes [7, 8, 9, 10, 11, 12, 13, 14]. By instantons in string theory we generally mean instantons which are geometrically realized as Euclidean extended objects wrapped on some non-trivial cycles of the geometry. Thus, in a sense, a stringy instanton has a "life of its own", not requiring an underlying gauge theory. This opens up the possibility of having contributions originating from instantons that do not admit a standard gauge theory realization. We shall refer to these instantons as *exotic*.

There has been some debate in the recent literature about the instances where such exotic instantons can actually contribute to the gauge theory superpotential in a non-trivial manner. In this work we will contribute to such a debate by considering backgrounds where a simple CFT description is possible, such as orbifolds or orientifolds thereof.

We present various simple examples of what we believe to be a rather generic situation. Namely, the presence of extra zero-modes for these instantons, in addition to those required by the counting of broken symmetries, makes some of their contributions vanish. Such extra zero-modes should not come as a surprise, since a D-brane instanton in a CY manifold breaks a total of four out of eight supercharges, i.e. it has two extra fermionic zero-modes from the point of view of holomorphic $\mathcal{N} = 1$ gauge theory quantities. We give some arguments as to why the backreaction of the space-filling branes on the geometry might not help in lifting these extra zero-modes. We further argue that only more radical changes of the background, such as the introduction of fluxes, deformations of the CY geometry or the introduction of orientifold planes, can remove these zero-modes. When this happens, exotic instantons do contribute to the gauge theory superpotential and may provide qualitative changes in the low energy effective dynamics, as for instance the stabilization of otherwise runaway directions.

We will be interested in Euclidean D-branes in type II theories. We will work with IIB fractional branes at orbifold and orientifold singularities rather than type IIA wrapped branes. The motivation for this choice of setting is two-fold. First, recent advances in the gauge/gravity correspondence require the study of exotic instantons, whose effects tend to stabilize the gauge theory rather than unstabilize it [15, 16, 9, 17], and the gauge/gravity correspondence is more naturally defined in the context of IIB theory. Second, similar effects are used in string phenomenology to try to understand possible mechanisms for neutrino masses [7, 8, 13]. This latest activity is mainly done in the type IIA scenario, but we find it easier to address some subtle issues in the IIB orbifold case.

While working in an exact string background, our considerations will nonetheless be only local, *i.e.* we will not be concerned with global issues such as tadpole cancellation that arise in proper compactifications. This is perfectly acceptable in the context of the gauge/gravity correspondence where the internal manifold is non-compact but, even for string phenomenology, the results we obtain stand (locally) when properly embedded in a consistent compactification.

The paper is organized as follows: In section 2 we set up the notation and discuss some preliminary material. In section 3 we discuss our first case, namely the $\mathcal{N} = 1$ $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold. After briefly recovering the usual instanton generated corrections to the superpotential we discuss the possible presence of additional exotic contributions

and find that they are not present because of the additional zero-modes. We conclude by giving a CFT argument on why such zero-modes are not expected to be lifted even by taking into account the backreaction of the D-branes, unless one is willing to move out the orbifold point in the CY moduli space. Sections 4 and 5 present two separate instances where exotic contributions are present after having removed the extra zero-modes by orientifolding. The first is an $\mathcal{N}=1$ orientifold, the second is an $\mathcal{N}=2$ orientifold, displaying corrections to the superpotential and the prepotential, respectively. We end with some conclusions and a discussion of further developments.

2. Preliminaries

In this section we briefly review the generic setup in the well understood $\mathcal{N}=4$ situation in order to introduce the notation for the various fields and moduli and their couplings. The more interesting theories we will consider next will be suitable projections of the $\mathcal{N}=4$ theory. In fact, the exotic cases can all be reduced to orbifolds/orientifolds of this master case once the appropriate projections on the Chan-Paton factors are performed.

Since we are interested in instanton physics (for comprehensive reviews see [18] and the recent [19]) we will take the ten dimensional metric to be Euclidean. We consider a system where both D3-branes and D(-1)-branes (D-instantons) are present. To be definite, we take N D3's and k D-instantons [1].

Quite generically we can distinguish three separate open string sectors:

- The gauge sector, made of those open strings with both ends on a D3-brane. We assume the brane world-volumes are lying along the first four coordinates x^μ and are orthogonal to the last six x^a. The massless fields in this sector form an $\mathcal{N}=4$ SYM multiplet [22]. We denote the bosonic components by A_μ and X^a. Written in $\mathcal{N}=1$ language this multiplet is formed by a gauge superfield whose field strength is denoted by W_α and three chiral superfields $\Phi^{1,2,3}$. With a slight abuse of notation, the bosonic components of the chiral superfields will also be denoted by Φ, *i.e.* $\Phi^1 = X^4 + iX^5$ and so on. In $\mathcal{N}=2$ language we have instead a gauge superfield $\mathcal{A}$ and a hypermultiplet H, all in the adjoint representation. The low energy action of these fields is a four dimensional $\mathcal{N}=4$ gauge theory. All these fields are $N\times N$ matrices for a gauge group SU(N).

[1]These D3/D(-1) brane systems (and their orbifold projections) are very useful and efficient in studying instanton effects from a stringy perspective even in the presence of non-trivial closed string backgrounds, both of NS-NS type [20] and of R-R type [21].

- The neutral sector, which comprises the zero-modes of strings with both ends on the D-instantons. It is usually referred to as the neutral sector because these modes do not transform under the gauge group. The zero-modes are easily obtained by dimensionally reducing the maximally supersymmetric gauge theory to zero dimensions. We will use an ADHM [23] inspired notation [5, 6]. We denote the bosonic fields as a_μ and χ^a, where the distinction between the two is made by the presence of the D3-branes. The fermionic zero-modes are denoted by $M^{\alpha A}$ and $\lambda_{\dot\alpha A}$, where α and $\dot\alpha$ denote the (positive and negative) four dimensional chiralities and A is an SU(4) (fundamental or anti-fundamental) index denoting the chirality in the transverse six dimensions. The ten dimensional chirality of both fields is taken to be negative. In Euclidean space M and λ must be treated as independent. When needed, we will also introduce the triplet of auxiliary fields D^c, directly analogous to the four dimensional D, that can be used to express the various interactions in an easier form as we will see momentarily. All these fields are $k \times k$ matrices where k is the instanton number.

- The charged sector, comprising the zero-modes of strings stretching between a D3-brane and a D-instanton. For each pair of such branes we have two conjugate sectors distinguished by the orientation of the string. In the NS sector, where the world-sheet fermions have opposite modding as the bosons, we obtain a bosonic spinor $\omega_{\dot\alpha}$ in the first four directions where the GSO projection picks out the negative chirality. In the conjugate sector, we will get an independent bosonic spinor $\bar\omega_{\dot\alpha}$ of the same chirality. Similarly, in the R sector, after the GSO projection we obtain a pair of independent fermions (one for each conjugate sector) both in the fundamental of SU(4) which we denote by μ^A and $\bar\mu^A$. These fields are rectangular matrices $N \times k$ and $k \times N$.

The couplings of the fields in the gauge sector give rise to a four dimensional gauge theory. The instanton corrections to such a theory are obtained by constructing the Lagrangian describing the interaction of the gauge sector with the charged sector zero-modes while performing the integral over *all* zero-modes, both charged and neutral. A crucial point to notice and which will be important later is that while the neutral modes do not transform under the gauge group, their presence affects the integral because of their coupling to the charged sector.

The part of the interaction involving only the instanton moduli is well known from the ADHM construction and it is essentially the reduction of the interacting gauge Lagrangian for these modes in a specific limit where the Yukawa terms for λ and the quadratic term for D are scaled out (see [18, 6] for details). The final form of this part

of the interaction is:

$$
\begin{aligned}
S_1 = \mathrm{tr}\Big\{ &- [a_\mu, \chi^a]^2 + \chi^a \bar\omega_{\dot\alpha}\omega^{\dot\alpha}\chi_a + \frac{i}{2}(\bar\Sigma^a)_{AB}\bar\mu^A\mu^B\chi_a - \frac{i}{4}(\bar\Sigma^a)_{AB}M^{\alpha A}[\chi_a, M^B_\alpha] \\
&+ i\left(\bar\mu^A\omega_{\dot\alpha} + \bar\omega_{\dot\alpha}\mu^A + \sigma^\mu_{\beta\dot\alpha}[M^{\beta A}, a_\mu]\right)\lambda^{\dot\alpha}_A - iD^c\Big(\bar\omega^{\dot\alpha}(\tau^c)^{\dot\beta}_{\dot\alpha}\omega_{\dot\beta} + i\bar\eta^c_{\mu\nu}[a^\mu, a^\nu]\Big)\Big\} \qquad (2.1)
\end{aligned}
$$

where the sum over colors and instanton indices is understood. τ denotes the usual Pauli matrices, $\bar\eta$ (and η) the 't Hooft symbols and $\bar\Sigma$ (and Σ) are used to construct the six-dimensional gamma-matrices

$$
\Gamma^a = \begin{pmatrix} 0 & \Sigma^a \\ \bar\Sigma^a & 0 \end{pmatrix} . \qquad (2.2)
$$

The above interactions can all be understood in terms of string diagrams on a disk with open string vertex operators inserted at the boundary in the $\alpha' \to 0$ limit.

The interaction of the charged sector with the scalars of the gauge sector can be worked out in a similar way and yields

$$
S_2 = \mathrm{tr}\left\{\bar\omega_{\dot\alpha}X^a X_a \omega^{\dot\alpha} + \frac{i}{2}(\bar\Sigma^a)_{AB}\bar\mu^A X_a \mu^B\right\} . \qquad (2.3)
$$

Let us rewrite the above action in a way which will be more illuminating in the following sections. Since we will be mainly focusing on situations where we have $\mathcal{N} = 1$ supersymmetry, it is useful to write explicitly all indices in SU(4) notation, and then break them into SU(3) representations. We thus write the six scalars X_a as the antisymmetric representation of SU(4) as follows

$$
X_{AB} = -X_{BA} \equiv (\bar\Sigma^a)_{AB}X_a . \qquad (2.4)
$$

The action S_2 then reads

$$
S_2 = \mathrm{tr}\left\{\frac{1}{8}\,\epsilon^{ABCD}\bar\omega_{\dot\alpha}X_{AB}X_{CD}\omega^{\dot\alpha} + \frac{i}{2}\,\bar\mu^A X_{AB}\mu^B\right\} . \qquad (2.5)
$$

Splitting now the indices A into $i = 1\ldots 3$ and 4, we can identify $\Phi^\dagger_i \equiv X_{i4}$ in the $\bar{\mathbf{3}}$ of SU(3) and $\Phi^i \equiv \frac{1}{2}\epsilon^{ijk}X_{jk}$ in the $\mathbf{3}$ of SU(3). Thus we can rewrite the action (2.5) as

$$
S_2 = \mathrm{tr}\left\{\frac{1}{2}\,\bar\omega_{\dot\alpha}\{\Phi^i, \Phi^\dagger_i\}\omega^{\dot\alpha} + \frac{i}{2}\,\bar\mu^i\Phi^\dagger_i\mu^4 - \frac{i}{2}\,\bar\mu^4\Phi^\dagger_i\mu^i - \frac{i}{2}\,\epsilon_{ijk}\bar\mu^i\Phi^j\mu^k\right\} . \qquad (2.6)
$$

In the above form, it is clear which zero-modes couple to the holomorphic superfields and which others couple to the anti-holomorphic ones. This distinction will play an important role later.

The main object of our investigation is the integral of $e^{-S_1-S_2}$ over *all* moduli

$$Z = \mathcal{C} \int d\{a, \chi, M, \lambda, D, \omega, \bar{\omega}, \mu, \bar{\mu}\}\, e^{-S_1-S_2} \ , \tag{2.7}$$

where we have lumped all field independent normalization constants (including the instanton classical action and the appropriate powers of α' required by dimensional analysis) into an overall coefficient $\mathcal{C}$. There are, of course, other interactions involving the fermions and the gauge bosons but, as far as the determination of the holomorphic quantities are concerned, they can be obtained from the previous ones and supersymmetry arguments. For example, a term in the superpotential is written as the integral over chiral superspace $\int dx^4 d\theta^2$ of a holomorphic function of the chiral superfields, but such a function is completely specified by its value for bosonic arguments at $\theta = 0$. Thus, if we can "factor out" a term $\int dx^4 d\theta^2$ from the moduli integral (2.7), whatever is left will define the complex function to be used in the superpotential and similarly for the prepotential in the $\mathcal{N} = 2$ case if we succeed in factoring out an integral over $\mathcal{N} = 2$ chiral superspace $\int dx^4 d\theta^4$.

The coordinates x and θ must of course come from the (super)translations broken by the instanton and they will be associated to the center of mass motion of the D-instanton, namely, $x^\mu = \mathrm{tr}\, a^\mu$ and $\theta^{\alpha A} = \mathrm{tr}\, M^{\alpha A}$ for some values of A.[2] One must pay attention however to the presence of possible additional neutral zero-modes coming either from the traceless parts of the above moduli or from the fields λ and χ. These modes must also be integrated over in (2.7) and their effects, as we shall see, can be quite dramatic. In particular, the presence of λ in some instances is crucial for the implementation of the usual ADHM fermionic constraints whereas in other circumstances it makes the whole contribution to the superpotential vanish. These extra λ zero-modes are ubiquitous in orbifold theories and generically make it difficult to obtain exotic instanton corrections for these models. As we shall see, they can however be easily projected out by an orientifold construction making the derivation of such terms possible.

In the full expression for the instanton corrections there will also be a field-independent normalization factor coming from the one-loop string diagrams and giving for instance the proper g_{YM} dependence in the case of the usual instanton corrections. In this paper we will only focus on the integral over the zero-modes, which gives the proper field-dependence, referring the reader to [10, 11] for a discussion of these other issues.

[2] Obviously, for the case of an anti-instanton, the roles of M and λ are reversed.

3. The $\mathcal{N}=1$ $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold

In order to present a concrete example of the above discussion, let us study a simple $\mathbf{C}^3/\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold singularity. The resulting $\mathcal{N}=1$ theory is a non-chiral four-node quiver gauge theory with matter in the bi-fundamental. Non-chirality implies that the four gauge group ranks can be chosen independently [24]. This corresponds to being able to find a basis of three independent fractional branes in the geometry (for a review on fractional branes on orbifolds see e.g. [25]).

The field content can be conveniently summarized in a quiver diagram, see Fig. 1, which, together with the cubic superpotential

$$\begin{aligned} W = \; & \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \Phi_{13}\Phi_{34}\Phi_{41} - \Phi_{14}\Phi_{43}\Phi_{31} \\ & + \Phi_{14}\Phi_{42}\Phi_{21} - \Phi_{12}\Phi_{24}\Phi_{41} + \Phi_{24}\Phi_{43}\Phi_{32} - \Phi_{23}\Phi_{34}\Phi_{42} \;, \end{aligned} \tag{3.1}$$

uniquely specifies the theory.

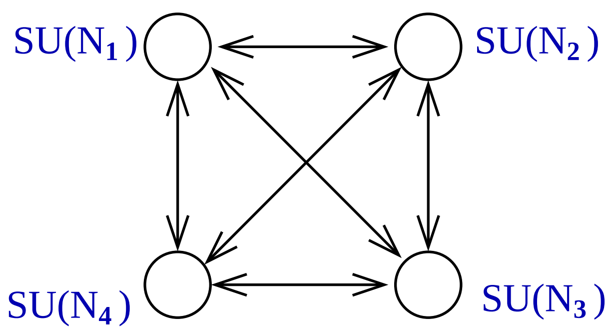

Figure 1: Quiver diagram for the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold theory. Round circles correspond to $\mathrm{SU}(N_\ell)$ gauge factors while the lines connecting quiver nodes represent the bi-fundamental chiral superfields $\Phi_{\ell m}$.

A stack of N regular D3-branes amounts to having one and the same rank assignment on the quiver. The gauge group is then $\mathrm{SU}(N)^4$ and the theory is an $\mathcal{N}=1$ SCFT. Fractional branes correspond instead to different (but anomlay free) rank assignments. Quite generically, fractional branes can be divided into three different classes, depending on the IR dynamics they trigger [26]. The non-chiral nature and the particularly symmetric structure of the orbifold under consideration allows one to easily construct any such instance of fractional brane class.

If we turn on a single node, we are left with a pure $\mathrm{SU}(N)$ SYM gauge theory, with no matter fields and no superpotential. This theory is believed to confine. The geometric dual effect is that the corresponding fractional brane leads to a geometric

transition where the branes disappear leaving behind a deformed geometry. Indeed, there is one such deformation in the above singularity.

Turning on two nodes leads already to more varied phenomena. There are now two bi-fundamental superfields, but still no tree level superpotential. Thus, the system is just like two coupled massless SQCD theories or, by a slightly asymmetric point of view, massless SQCD with a gauged diagonal flavor group. The low-energy behavior depends on the relative ranks of the two nodes.

If the ranks are different, the node with the highest rank is in a situation where it has less flavors than colors. Then an Affleck-Dine-Seiberg (ADS) superpotential [27, 28] should be dynamically generated, leading eventually to a runaway behavior. This set up of fractional branes is sometimes referred to as supersymmetry breaking fractional branes [29, 26, 30].

If the ranks are the same we are in a situation similar to $N_f = N_c$ SQCD for both nodes. Hence we expect to have a moduli space of SUSY vacua, which gets deformed, but not lifted, at the quantum level. This moduli space is roughly identified in the geometry with the fact that the relevant fractional branes are interpreted as D5-branes wrapped on the 2-cycle of a singularity which is locally $\mathbf{C} \times (\mathbf{C}^2/\mathbf{Z}_2)$. Such a fractional brane can move in the $\mathbf{C}$ direction. This is what is called an $\mathcal{N} = 2$ fractional brane since, at least geometrically, it resembles very much the situation of fractional branes at $\mathcal{N} = 2$ singularities.

In what follows we use the two-node example as a simple setting in which we can analyze the subtleties involved in the integration over the neutral modes. For the gauge theory instanton case it is known that there are *extra* neutral fermionic zero-modes in addition to those required to generate the superpotential. Their integration allows to recover the fermionic ADHM constraints on the moduli space of the usual field theory instantons. For such instantons, we will be able to obtain the ADS superpotential and corresponding runaway behavior in the familiar context with N_c and N_f fractional branes at the respective nodes, for $N_f = N_c - 1$. On the other hand, we will argue that the presence of such extra zero-modes rules out the possibility of having exotic instanton effects, such as terms involving baryonic operators in the $N_f = N_c$ case. It was the desire to study such possible contributions that constituted the original motivation for this investigation. We will first show that such effects are absent for this theory as it stands, and we will later discuss when and how this problem can be cured.[3]

[3]In a situation where the CFT description is less under control than in the setting discussed in the present paper, it has been argued in [17] that such baryonic couplings do arise in the context of fractional branes on orbifolds of the conifold, possibly at the expense of introducing O-planes. Also in a IIA set up similar to the ones of [7, 8, 10, 11, 13] it seems reasonable that one can wrap an ED2-brane along an O6-plane and produce such couplings on other intersecting D6-branes.

Our orbifold theory can be easily obtained as an orbifold projection of $\mathcal{N}=4$ SYM. The orbifolding procedure and the derivation of the superpotential (3.1) are by now standard. We briefly recall the main points in order to fix the notation and because some of the details will be useful later in describing the instantons in such a set up.

The group $\mathbf{Z}_2 \times \mathbf{Z}_2$ has four elements: the identity e, the generators of the two $\mathbf{Z}_2$ that we denote with g_1 and g_2 and their product, denoted by $g_3 = g_1 g_2$. If we introduce complex coordinates $(z_1, z_2, z_3) \in \mathbf{C}^3$

$$z^1 = x^4 + ix^5 \quad , \quad z^2 = x^6 + ix^7 \quad , \quad z^3 = x^8 + ix^9 \tag{3.2}$$

the action of the orbifold group can be defined as in Table 1.

	z^1	z^2	z^3
e	z^1	z^2	z^3
g_1	z^1	$-z^2$	$-z^3$
g_2	$-z^1$	z^2	$-z^3$
g_3	$-z^1$	$-z^2$	z^3

Table 1: The action of the orbifold generators.

Let $\gamma(g)$ be the regular representation of the orbifold group on the Chan-Paton factors. If the orbifold is abelian, as always in the cases we shall be interested in, we can always diagonalize all matrices $\gamma(g)$. We will assume that the two generators have the following matrix representation

$$\gamma(g_1) = \sigma_3 \otimes \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad , \quad \gamma(g_2) = \mathbf{1} \otimes \sigma_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3.3}$$

where the 1's denote $N_\ell \times N_\ell$ unit matrices ($\ell = 1, ..., 4$). Then, the orbifold projection amounts to enforcing the conditions

$$A_\mu = \gamma(g) A_\mu \gamma(g)^{-1} \quad , \quad \Phi^i = \pm \gamma(g) \Phi^i \gamma(g)^{-1} \tag{3.4}$$

where the sign $\pm$ must be chosen according to the action of the orbifold generators g that can be read off from Table 1. With the choice (3.3), the vector superfields are block diagonal matrices of different size (N_1, N_2, N_3, N_4), one for each node of the

quiver, while the three chiral superfields Φ^i have the following form [24]

$$\Phi^1 = \begin{pmatrix} 0 & \times & 0 & 0 \\ \times & 0 & 0 & 0 \\ 0 & 0 & 0 & \times \\ 0 & 0 & \times & 0 \end{pmatrix} , \quad \Phi^2 = \begin{pmatrix} 0 & 0 & \times & 0 \\ 0 & 0 & 0 & \times \\ \times & 0 & 0 & 0 \\ 0 & \times & 0 & 0 \end{pmatrix} , \quad \Phi^3 = \begin{pmatrix} 0 & 0 & 0 & \times \\ 0 & 0 & \times & 0 \\ 0 & \times & 0 & 0 \\ \times & 0 & 0 & 0 \end{pmatrix} , \tag{3.5}$$

where the crosses represent the non-zero entries $\Phi_{\ell m}$ appearing in the superpotential (3.1).

3.1 Instanton sector

Now consider D-instantons in the above set up. Such instantons preserve half of the 4 supercharges preserved by the system of D3-branes plus orbifold. In this respect recall that the fractional branes preserve exactly the same supercharges as the regular branes.[4] Using the $\mathcal{N} = 4$ construction of the previous section and the structure of the orbifold presented in eq. (3.5), we now proceed in describing the zero-modes for such instantons.

The neutral sector is very similar to the gauge sector. Indeed, in the (-1) superghost picture, the vertex operators for such strings will be exactly the same, except for the $e^{ip\cdot X}$ factor which is absent for the instanton. The Chan-Paton structure will also be the same, so that the same pattern of fractional D-instantons will arise as for the fractional D3-branes. In particular, the only regular D-instanton (which could be thought of as deriving from the one of $\mathcal{N} = 4$ SYM) is the one with rank (instanton number) one at every node. All other situations can be thought of as fractional D-instantons, which can be interpreted as Euclidean D1-branes wrapped on the two-cycles at the singularity, ED1 for short. Generically, we can then characterize an instanton configuration in our orbifold by (k_1, k_2, k_3, k_4).

Following the notation introduced in section 2, the bosonic modes will comprise a 4×4 block diagonal matrix a^μ, and six more matrix fields $\chi^1, \dots \chi^6$, that can be paired into three complex matrix fields $\chi^1+i\chi^2, \chi^3+i\chi^4, \chi^5+i\chi^6$, having the same structure as (3.5) but now where each block entry is a $k_\ell \times k_m$ matrix. On the fermionic zero-modes $M^{\alpha A}$ and $\lambda_{\dot\alpha A}$ (also matrices) the orbifold projection enforces the conditions

$$M^{\alpha A} = R(g)^A{}_B\, \gamma(g) M^{\alpha B} \gamma(g)^{-1} \quad , \quad \lambda_{\dot\alpha A} = \gamma(g) \lambda_{\dot\alpha B} \gamma(g)^{-1} R(g)^B{}_A \tag{3.6}$$

[4] There is another Euclidean brane which preserves two supercharges, namely the Euclidean (anti) D3-branes orthogonal to the 4 dimensions of space-time. We will be considering here only the D-instantons, leaving the complete analysis of the other effects to future work. In this context, note that the extended brane instantons would have an infinite action (and thus a vanishing contribution) in the strict non-compact set up we are using here.

where $R(g)$ is the orbifold action of Table 1 in the spinor representation which can be chosen as

$$R(g_1) = -\Gamma^{6789} \quad , \quad R(g_2) = -\Gamma^{4589} \; . \tag{3.7}$$

It is easy to find an explicit representation of the Dirac matrices such that $M^{\alpha A}$ and $\lambda_{\dot{\alpha}A}$ for $A = 1, 2, 3$ also have the structure of (3.5) while for $A = 4$ they are block diagonal. Equivalently, one could write the spinor indices in the internal space in terms of the three SO(2) charges associated to the embedding SO(2) × SO(2) × SO(2) ⊂ SO(6) ≃ SU(4)

$$\begin{aligned} &M^{\alpha-++} = M^{\alpha 1} \; , \; M^{\alpha+-+} = M^{\alpha 2} \; , \; M^{\alpha++-} = M^{\alpha 3} \; , \; M^{\alpha---} = M^{\alpha 4} \; , \\ &\lambda_{\dot{\alpha}+--} = \lambda_{\dot{\alpha}1} \; , \; \lambda_{\dot{\alpha}-+-} = \lambda_{\dot{\alpha}2} \; , \; \lambda_{\dot{\alpha}--+} = \lambda_{\dot{\alpha}3} \; , \; \lambda_{\dot{\alpha}+++} = \lambda_{\dot{\alpha}4} \; . \end{aligned} \tag{3.8}$$

The most notable difference between the neutral sector and the gauge theory on the D3-branes is that, whereas in the four-dimensional theory the $U(1)$ gauge factors are rendered massive by a generalization of the Green-Schwarz mechanism and do not appear in the low energy action, for the instanton they are in fact present and enter crucially into the dynamics.

Let us finally turn to the charged sector, describing strings going from the instantons to the D3-branes. The analysis of the spectrum and the action of the orbifold group on the Chan-Paton factors show, in particular, that the bosonic zero-modes are diagonal in the gauge factors. There are four block diagonal matrices of bosonic zero-modes $\omega_{\dot{\alpha}}$, $\bar{\omega}_{\dot{\alpha}}$ with entries $N_\ell \times k_\ell$ and $k_\ell \times N_\ell$ respectively and eight fermionic matrices μ^A, $\bar{\mu}^A$ with entries $N_\ell \times k_m$ and $k_m \times N_\ell$, that again display the same structure as above – same as (3.5) for $A = 1, 2, 3$ and diagonal for $A = 4$.

3.2 Recovery of the ADS superpotential

The measure on the moduli space of the instantons and the ADHM constraints are simply obtained by inserting the above expressions into the moduli integral (2.7). If one chooses some of the N_ℓ or k_ℓ to vanish one can deduce immediately from the structure of the projection which modes will survive and which will not.

As a consistency check, one can try to reproduce the ADS correction to the superpotential [27, 28] for the theory with two nodes. Take fractional branes corresponding to a rank assignment $(N_c, N_f, 0, 0)$, and consider the effect of a ED1 corresponding to instanton numbers $(1, 0, 0, 0)$.

The only chiral fields present are the two components of Φ^1 connecting the first

and second node

$$\Phi^1 = \begin{pmatrix} 0 & Q & 0 & 0 \\ \tilde{Q} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \ . \tag{3.9}$$

Since the instanton is sitting only at one node, all off diagonal neutral modes are absent, as they connect instantons at two distinct nodes. Thus, the only massless modes present in the neutral sector are four bosons x^μ, denoting the upper-left component of a^μ, two fermions θ^α denoting the upper-left component of $M^{\alpha 4}$ and two more fermions $\lambda_{\dot\alpha}$ denoting the upper-left component of $\lambda_{\dot\alpha 4}$. We have identified the non zero entries of a^μ and $M^{\alpha 4}$ with the super-coordinates x^μ and θ^α since they precisely correspond to the Goldstone modes of the super-translation symmetries broken by the instanton and do not appear in $S_1 + S_2$ (cfr. (2.1) and (2.3)). Their integration produces the integral over space-time and half of Grassmann space which precedes the superpotential term to which the instanton contributes. On the contrary, $\lambda_{\dot\alpha}$ appears in S_1 and when it is integrated it yields the fermionic ADHM constraint.

In the charged sector, we have bosonic zero-modes $\omega^u_{\dot\alpha}$ and $\bar\omega_{\dot\alpha u}$, with u an index in the fundamental or anti-fundamental of $\mathrm{SU}(N_c)$. In addition, there are fermionic zero-modes μ^u and $\bar\mu_u$ with indices in $\mathrm{SU}(N_c)$, together with additional fermionic zero-modes μ'^f and $\bar\mu'_f$ where the index f is now in the fundamental or anti-fundamental of $\mathrm{SU}(N_f)$.[5] Note that the μ zero-modes carry an SU(4) index 4 (being on the diagonal) while the μ' zero-modes carry an SU(4) index 1, since they are of the same form as Φ^1.

All this can be conveniently summarized in a generalized quiver diagram as represented in Fig. 2, which accounts for both the brane configuration and the instanton zero-modes.

For a single instanton, the action (2.1) greatly simplifies since many fields are vanishing as well as all commutators and one gets

$$S_1 = i\,(\bar\mu_u \omega^u_{\dot\alpha} + \bar\omega_{\dot\alpha u}\mu^u)\,\lambda^{\dot\alpha} - i D^c \bar\omega^{\dot\alpha}_u (\tau^c)^{\dot\beta}_{\dot\alpha}\omega^u_{\dot\beta} \ . \tag{3.10}$$

Similarly, the coupling of the charged modes to the chiral superfield can be expressed by writing eq. (2.3) as

$$S_2 = \frac{1}{2}\,\bar\omega_{\dot\alpha u}\big(Q^u_f Q^{\dagger f}_{\ \ v} + \tilde{Q}^{\dagger u}_{\ \ f}\tilde{Q}^f_v\big)\omega^{\dot\alpha v} - \frac{i}{2}\,\bar\mu_u \tilde{Q}^{\dagger u}_{\ \ f}\mu'^f + \frac{i}{2}\,\bar\mu'_f Q^{\dagger f}_{\ \ u}\mu^u \ . \tag{3.11}$$

Note that it is the anti-holomorphic superfields that enter in the couplings with the fermionic zero-modes, as is clear by comparing with (2.6). The above action is exactly the same which appears in the ADHM construction as reviewed in [18].

[5] Recall that the bosonic zero-modes are diagonal in the gauge factors; therefore there are no $\omega^f_{\dot\alpha}$ and $\bar\omega_{\dot\alpha f}$ zero-modes.

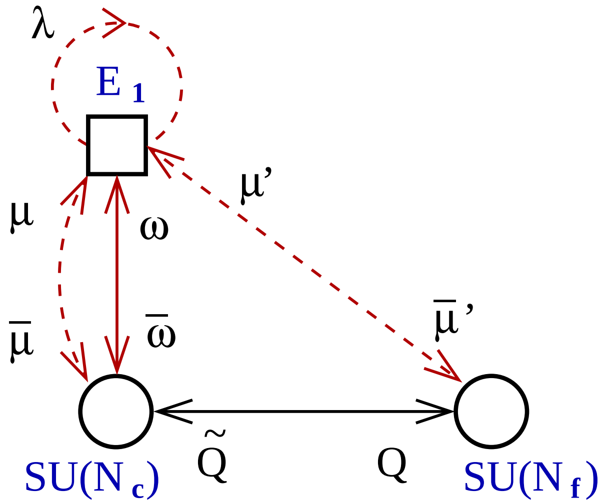

Figure 2: Quiver diagram describing an ordinary instanton in a $\mathrm{SU}(N_c) \times \mathrm{SU}(N_f)$ theory. Gauge theory nodes are represented by round circles, instanton nodes by squares. The ED1 is wrapped on the same cycle as the color branes. All zero-modes are included except the θ's and the x^μ's, which only contribute to the measure for the integral over chiral superspace.

We are now ready to perform the integral (2.7) over all the existing zero-modes. Writing

$$Z = \int dx^4 d\theta^2 \, W \ , \tag{3.12}$$

we see that the instanton induced superpotential is

$$W = \mathcal{C} \int d\{\lambda, D, \omega, \bar{\omega}, \mu, \bar{\mu}\} \, e^{-S_1 - S_2} \ . \tag{3.13}$$

The integrals over D and λ enforce the bosonic and fermionic ADHM constraints, respectively. Thus

$$W = \mathcal{C} \int d\{\omega, \bar{\omega}, \mu, \bar{\mu}\} \, \delta(\bar{\mu}_u \omega^u_{\dot{\alpha}} + \bar{\omega}_{\dot{\alpha} u} \mu^u) \, \delta(\bar{\omega}^{\dot{\alpha}}_u (\tau^c)^{\dot{\beta}}_{\dot{\alpha}} \omega^u_{\dot{\beta}}) \, e^{-S_2} \ . \tag{3.14}$$

We essentially arrive at the point of having to evaluate an integral over a set of zero-modes which is exactly the same as the one discussed in detail in the literature, *e.g.* [18]. We thus quickly go to the result referring the reader to the above review for further details. First of all, it is easy to see that, due to the presence of extra μ modes in the integrand from the fermionic delta function, only when $N_f = N_c - 1$ we obtain a non-vanishing result. After having integrated over the μ and μ', we are left with a (constrained) gaussian integration that can be performed *e.g.* by going to a region

of the moduli space where the chiral fields are diagonal, up to a row/column of zeroes. Furthermore, the D-terms in the gauge sector constrain the quark superfields to obey $QQ^\dagger = \tilde{Q}^\dagger \tilde{Q}$, so that the bosonic integration brings the square of a simple determinant in the denominator. The last fermionic integration conspires to cancel the anti-holomorphic contributions and gives

$$W_{ADS} = \frac{\Lambda^{2N_c+1}}{\det(\tilde{Q}Q)} \ , \tag{3.15}$$

which is just the expected ADS superpotential for $N_f = N_c - 1$, the only case where such non-perturbative contribution is generated by a genuine one-instanton effect and not by gaugino condensation. In (3.15) Λ is the SQCD strong coupling scale that is reconstructed by the combination of $e^{-8\pi^2/g^2}$ coming from the instanton action with various dimensional factors coming from the normalization of the instanton measure [18].

3.3 Absence of exotic contributions

Until now, we have reproduced from stringy considerations the effect that is supposed to be generated also by instantons in the gauge theory. Considering a slightly different set up, we would like to study the possibility of generating other terms.

Let us consider a system with rank assignment $(N_c, N_f, 0, 0)$, as before, but fractional instanton numbers $(0, 0, 1, 0)$. In other words, we study the effect of a single fractional instanton sitting on an *unoccupied* node of the gauge theory. The quiver diagram, with the relevant zero-modes structure, is given in Fig. 3.

The neutral zero-modes of the instanton sector are the same as before. This is because the quantization of this sector does not know the whereabouts of the D3-branes and thus all nodes are equivalent, in this respect. In the mixed sector, we have no bosonic zero-modes now, since the ω and $\bar{\omega}$ are diagonal. Note that, although we always have four mixed (ND) boundary conditions, due to the quiver structure induced by the orbifold, here we effectively realize the same situation one has when there are eight ND directions, namely that the bosonic sector of the charged moduli is empty.

On the other hand, there are fermionic zero-modes μ^u, $\bar{\mu}_u$, μ'^f and $\bar{\mu}'_f$, as in the previous case. Note that despite having the same name, these zero-modes correspond actually to different Chan-Paton matrix elements with respect to the previous ones, the difference being in the instanton index that is not written explicitly. In particular we can think of μ and μ' as carrying an SU(4) index 2 and 3 respectively.

Because of the absence of bosonic charged modes, the action (2.1) is identically

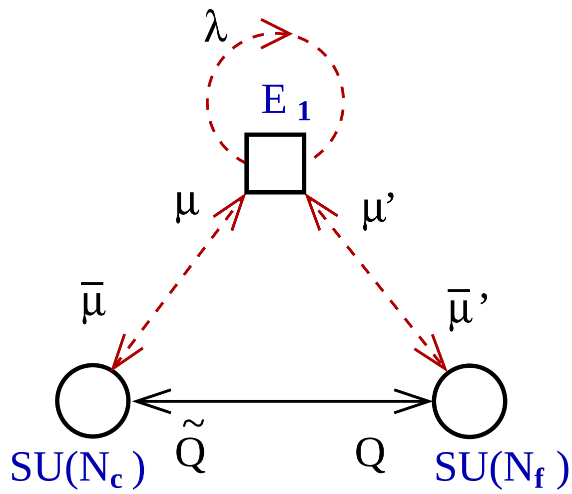

Figure 3: Quiver diagram describing an exotic instanton in a $\mathrm{SU}(N_c) \times \mathrm{SU}(N_f)$ theory. Gauge theory nodes are represented by round circles, instanton nodes by squares. The ED1 is wrapped on a different cycle with respect to both sets of quiver branes.

zero and the action (2.3) contains only the last term:

$$\begin{aligned} S_1 &= 0 \\ S_2 &= \frac{i}{2}\,\bar{\mu}_u Q^u_f \mu'^f - \frac{i}{2}\,\bar{\mu}'_f \tilde{Q}^f_u \mu^u . \end{aligned} \tag{3.16}$$

Note that in this case it is the holomorphic superfields which appear above, as is clear from (2.6) and from noticing that the diagonal fermionic zero-mode μ^4 is not present. We are thus led to consider

$$W = \mathcal{C} \int d\{\lambda, D, \mu, \bar{\mu}\}\, e^{-S_2} \ . \tag{3.17}$$

One notices right away that the integral over the charged modes is non vanishing (only) for the case $N_f = N_c$ and gives a tantalizing contribution proportional to $B\tilde{B}$, where $B = \det Q$ and $\tilde{B} = \det \tilde{Q}$ are the baryon fields of the theory. However, we must carefully analyze the integration over the remaining zero-modes of the neutral sector. Now neither D nor λ appear in the integrand. The integral over D does not raise any concern: it is, after all, an auxiliary field and its disappearance from the integrand is due to the peculiarities of the ADHM limit. Before taking this limit, D appeared quadratically in the action and could be integrated out, leaving an overall normalization constant. The integral over λ is another issue. In this case, λ is absent from the integrand even before taking the ADHM limit and its integration multiplies

the above result by zero, making the overall contribution of such instantons to the superpotential vanishing. Of course, the presence of such extra zero-modes should not come as a surprise since they correspond to the two extra broken supersymmetries of an instanton on a CY.

Therefore we see that the neutral zero-modes contribution, in the exotic instanton case, plays a dramatic role and conspires to make everything vanishing (as opposite to the ADS case analyzed before). A natural question is to see whether these zero-modes get lifted by some effect we have not taken into account, yet. For one thing, supersymmetry arguments would make one think that taking into account the back-reaction of the D3-branes might change things. However, in the following subsection we show that this seems not to be the case.

3.4 Study of the back-reaction

Let us stick to the case $N_f = N_c$, which is the only one where the integral (3.17) might give a non-vanishing contribution. In this case the fractional brane system is nothing but a stack of (N_c) $\mathcal{N} = 2$ fractional branes. These branes couple to only one of the 3 closed string twisted sectors [24]. More specifically, they source the metric $h_{\mu\nu}$, the R-R four-form potential $C_{\mu\nu\rho\sigma}$ and two twisted scalars b and c from the NS-NS and R-R sector respectively. This means that the disk one-point function of their vertex operators [31, 32] is non vanishing when the disk boundary is attached to such D3-branes. (Indeed in this way or, equivalently, by using the boundary-state formalism [33, 34], one can derive the profile for these fields.)

If the back-reaction of these fields on the instanton lifted the extra zero-modes λ's, this should be visible when computing the one point function of the corresponding closed string vertex operators on a disk with insertions on this boundary of the vertex operators for such moduli. To see whether such coupling is there, we first need to write down the vertex operators for the λ's in the ($\pm 1/2$) superghost pictures. The vertex in the ($-1/2$) picture is found *e.g.* in [6] and reads

$$V_\lambda^{-1/2}(z) = \lambda_{\dot\alpha A} S^{\dot\alpha}(z) S^A(z) e^{-\phi(z)/2} \ , \tag{3.18}$$

where $S^{\dot\alpha}(z)$ and $S^A(z)$ are the spin-fields in the first four and last six directions respectively. For our argument we need to focus on the $S^A(z)$ dependence. Since the modulus that survives the orbifold projection is, with our conventions, $\lambda_{\dot\alpha 4} = \lambda_{\dot\alpha +++}$, we write the corresponding spin-field as

$$S^{+++}(z) = e^{iH_1(z)/2} e^{iH_2(z)/2} e^{iH_3(z)/2}, \tag{3.19}$$

where $H_i(z)$ is the free boson used to bosonize the fermionic sector in the i-th complex direction: $\psi^i(z) = e^{iH_i(z)}$. The vertex operator in the $+1/2$ picture can be obtained by

applying the picture-changing operator to (3.18)

$$V_\lambda^{1/2}(z) = [Q_{\rm BRST}, \xi V_\lambda^{-1/2}(z)] \ . \tag{3.20}$$

The crucial part in $Q_{\rm BRST}$ is [31]

$$Q_{\rm BRST} = \oint \frac{dz}{2\pi i}\, \eta\, e^{\phi} \left(\psi^\mu \partial X^\mu + \bar\psi^i \partial Z^i + \psi^i \partial \bar Z^i\right) + \ldots \tag{3.21}$$

Because of the nature of the supercurrent, we see that (3.21) flips at most one sign in (3.19), hence the product $V_\lambda^{-1/2} V_\lambda^{1/2}$ will always carry an unbalanced charge in some of the three internal SO(2) groups. On the other hand, the vertex operators for the fields sourced by the fractional D3's cannot compensate such an unbalance. Hence, their correlation function on the D-instanton with the insertion of $V_\lambda^{-1/2} V_\lambda^{1/2}$ carries a charge unbalance and therefore vanishes. Therefore, at least within the above perturbative approach, the neutral zero-modes seem not to get lifted by the back-reaction of the D3-branes.

One might consider some additional ingredients which could provide the lifting. A natural guess would be moving in the CY moduli space or adding suitable background fluxes [35, 36]. There are indeed non-vanishing background fields at the orbifold point, *i.e.* the b fields of the twisted sectors which the $\mathcal{N} = 2$ fractional branes do not couple to. These fields, however, being not associated to geometric deformations of the internal space should be described by a CFT vertex operator uncharged under the SO(2)'s, simply because of Lorentz invariance in the internal space. Therefore, the only way to get an effective mass term for the zero-modes λ would be to move out of the orbifold point in the CY moduli space. Indeed, the other moduli of the NS-NS twisted sector, being associated to geometric blow-ups of the singularity, are charged under (some of) the internal SO(2)'s and can have a non vanishing coupling with the λ's. More generically, complicated closed string background fluxes might be suitable. This is an interesting option which however we do not pursue here, since we want to stick to situations where a CFT description is available.

A more radical thing to do is to remove the zero-modes from the very start, for instance by means of an orientifold projection [37, 38]. This is the option we are going to consider in the remainder of this work.

4. The $\mathcal{N} = 1$ $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold

In this section we supplement our orbifold background by an O3 orientifold and show that in this case exotic instanton contributions do arise and provide new terms in the

superpotential. We refer to *e.g.* [39, 40, 41] for a comprehensive discussion of $\mathcal{N}=1$ and $\mathcal{N}=2$ orientifolds.

The first ingredient we need is the action of the O3-plane on the various fields. Denote by Ω the generator of the orientifold. The action of Ω on the vertex operators for the various fields (ignoring for the time being the Chan-Paton factors) is well known. The vertex operators for the bosonic fields on the D3-brane contain, in the 0 picture, the following terms: $A_\mu \sim \partial_\tau x^\mu$ and $\Phi^i \sim \partial_\sigma \bar{z}^i$. They both change sign under Ω, the first because of the derivative ∂_τ and the second because the orientifold action for the O3-plane is always accompanied by a simultaneous reflection of all the transverse coordinates z^i.

The action of the orientifold on the Chan-Paton factors is realized by means of a matrix $\gamma(\Omega)$ which in presence of an orbifold must satisfy the following consistency condition [39]

$$\gamma(g)\gamma(\Omega)\gamma(g)^T = +\gamma(\Omega) \tag{4.1}$$

for all orbifold generators g. This amounts to require that the orientifold projection commutes with the orbifold projection. The matrix $\gamma(\Omega)$ can be either symmetric or anti-symmetric. We choose to perform an anti-symmetric orientifold projection on the D3 branes and denote the corresponding matrix by $\gamma_-(\Omega)$. This requires having an even number N_ℓ of D3 branes on each node of the quiver so that we can write

$$\gamma_-(\Omega) = \begin{pmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & 0 \\ 0 & 0 & \epsilon_3 & 0 \\ 0 & 0 & 0 & \epsilon_4 \end{pmatrix} \tag{4.2}$$

where the ϵ_ℓ's are $N_\ell \times N_\ell$ antisymmetric matrices obeying $\epsilon_\ell^2 = -1$. Using (3.3) and (4.2) it is straightforward to verify that the consistency condition (4.1) is verified.

The field content of the stacks of fractional D3-branes in this orientifold model is obtained by supplementing the orbifold conditions (3.4) with the orientifold ones

$$A_\mu = -\gamma_-(\Omega) A_\mu^T \gamma_-(\Omega)^{-1} \quad , \quad \Phi^l = -\gamma_-(\Omega)\Phi^{lT}\gamma_-(\Omega)^{-1}. \tag{4.3}$$

This implies that $A_\mu = \mathrm{diag}\,(A_\mu^1, A_\mu^2, A_\mu^3, A_\mu^4)$ with $A_\mu^\ell = \epsilon_\ell A_\mu^{iT} \epsilon_\ell$. Thus, the resulting gauge theory is a $\mathrm{USp}(N_1) \times \mathrm{USp}(N_2) \times \mathrm{USp}(N_3) \times \mathrm{USp}(N_4)$ theory. The chiral superfields, which after the orbifold have the structure (3.5), are such that the $\Phi_{\ell m}$ component joining the nodes ℓ and m of the quiver, must obey the orientifold condition $\Phi_{\ell m} = \epsilon_\ell \Phi_{m\ell}^T \epsilon_m$. In the following, we will take $N_3 = N_4 = 0$ so that we are left with only two gauge groups and no tree level superpotential.

4.1 Instanton sector

Let us now consider the instanton sector, starting by analyzing the zero-mode content in the neutral sector. There are two basic changes to the previous story. The first is that the vertex operator for a_μ is now proportional to $\partial_\sigma x^\mu$, not to $\partial_\tau x^\mu$ and it remains invariant under Ω (the vertex operator for χ_a still changes sign). The second is that the crucial consistency condition discussed in [38] requires that we now represent the action of Ω on the Chan-Paton factors of the neutral modes by a symmetric matrix which can be taken to be

$$\gamma_+(\Omega) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \tag{4.4}$$

where the 1's are $k_\ell \times k_\ell$ unit matrices. The matrix a_μ will be 4×4 block diagonal, *e.g.* $a_\mu = \mathrm{diag}\,(a^1_\mu, a^2_\mu, a^3_\mu, a^4_\mu)$, but now $a^\ell_\mu = a^{\ell T}_\mu$. The most generic situation is to have a configuration with instanton numbers (k_1, k_2, k_3, k_4). By considering a configuration with $k_3 = 1$ and $k_1 = k_2 = k_4 = 0$, we can project out all bosonic zero-modes except for the four components a^3_μ that we denote by x_μ. The scalars $\chi^4 \ldots \chi^9$ are off-diagonal and we shall not consider them further.

The nice surprise comes when considering the orientifold action on the fermionic neutral zero-modes $M^{\alpha A}$ and $\lambda_{\dot\alpha A}$. The orbifold part of the group acts on the spinor indices as in (3.7), while the orientifold projection acts as the reflection in the transverse space, namely

$$R(\Omega) = -i\,\Gamma^{456789} \tag{4.5}$$

Putting together the orbifold projections (3.6) with the orientifold ones

$$M^{\alpha A} = R^A{}_B(\Omega)\gamma_+(\Omega)(M^{\alpha B})^T\gamma_+(\Omega)^{-1} \quad , \quad \lambda_{\dot\alpha A} = \gamma_+(\Omega)(\lambda_{\dot\alpha B})^T\gamma_+(\Omega)^{-1}R^B{}_A(\Omega) \tag{4.6}$$

we can find the spectrum of surviving fermionic zero-modes. Using (4.4) and (4.5), it is easy to see that (4.6) implies

$$M^{\alpha A} = (M^{\alpha A})^T \quad , \quad \lambda_{\dot\alpha A} = -(\lambda_{\dot\alpha A})^T \; . \tag{4.7}$$

Thus, for the simple case where $k_3 = 1$ and $k_1 = k_2 = k_4 = 0$, all λ's are projected out and only *two* chiral M zero-modes remain: $M^{\alpha---}$, to be identified with the $\mathcal{N} = 1$ chiral superspace coordinates θ^α.

Also the charged zero-modes are easy to discuss in this simple scenario. There are no bosonic modes since the D-instanton and the D3-branes sit at different nodes while

the bosonic modes are necessarily diagonal. Most of the fermionic zero-modes μ^A and $\bar{\mu}^A$ are also projected out by the orbifold condition

$$\mu^A = R(g)^A{}_B \gamma(g) \mu^B \gamma(g)^{-1} \quad , \quad \bar{\mu}^A = R(g)^A{}_B \gamma(g) \bar{\mu}^B \gamma(g)^{-1} \ . \tag{4.8}$$

Finally, the orientifold condition relates this time the fields in the conjugate sectors, allowing one to express $\bar{\mu}$ as a linear combination of the μ

$$\bar{\mu}^A = R(\Omega)^A{}_B \gamma_+(\Omega) (\mu^B)^T \gamma_-(\Omega)^{-1} \ . \tag{4.9}$$

The only charged modes surviving these projections can be expressed, in block 4×4 notation, as

$$\mu^2 = \begin{pmatrix} 0 & 0 & \mu_{13} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \quad \bar{\mu}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{\mu}_{31} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ,$$
$$\mu^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \mu_{23} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \quad \bar{\mu}^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \bar{\mu}_{32} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{4.10}$$

where the entries, to be thought of as column/row vectors in the fundamental/anti-fundamental of $\mathrm{SU}(N_\ell)$ depending on their position, are such that $\bar{\mu}_{31} = -\mu_{13}^T \epsilon_1$ and $\bar{\mu}_{32} = -\mu_{23}^T \epsilon_2$.

Thus, in the case where we have fractional D3 branes $(N_1, N_2, 0, 0)$ and an exotic instanton $(0, 0, 1, 0)$, the only surviving chiral field is $\Phi_{12} \equiv \epsilon_1 \Phi_{21}^T \epsilon_2$, the orientifold projection eliminates the offending λ's and we are left with just the neutral zero-modes x_μ and θ^α and the charged ones μ_{13} and μ_{23}. This is summarized in the generalized quiver of Fig. 4.

In this case the instanton partition function is

$$Z = \int dx^4 d\theta^2 \ W \tag{4.11}$$

where the superpotential W is

$$W = \mathcal{C} \int d\mu \ e^{-S_1 - S_2} = \mathcal{C} \int d\mu_{13} d\mu_{23} \ e^{i \mu_{13}^T \epsilon_1 \Phi_{12} \mu_{23}} \ . \tag{4.12}$$

This integral clearly vanishes unless $N_1 = N_2$, in which case we have

$$W \ \propto \ \det(\Phi_{12}) \tag{4.13}$$

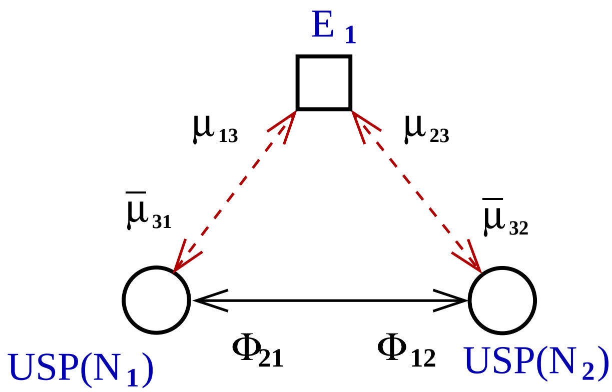

Figure 4: The generalized $\mathbf{Z}_2 \times \mathbf{Z}_2$ orientifold quiver and the exotic instanton contribution.

We thus see that exotic instanton corrections are possible in this simple model.[6]

It is interesting to note that the above correction is present in the same case $(N_1 = N_2 \equiv N)$ where the usual ADS superpotential for $\mathrm{USp}(N)$ is generated [42]

$$W_{ADS} = \frac{\Lambda^{2N+3}}{\det(\Phi_{12})} \tag{4.14}$$

and its presence stabilizes the runaway behavior and gives a theory with a non-trivial moduli space of supersymmetric vacua given by $\det(\Phi_{12}) = \text{const}$. Of course, the ADS superpotential for this case can also be constructed along the same lines as section 3.2, see e.g. [18]. In fact, this derivation is somewhat simpler than the one for the $SU(N)$ gauge group since there are no ADHM constraints at all in the one instanton case.

We think the above situation is not specific to the background we have been considering, but is in fact quite generic. As soon as the λ zero-modes are consistently lifted, we expect the exotic instantons to contribute new superpotential terms. As a further example, in the next section we will consider a $\mathcal{N} = 2$ model, where exotic instantons will turn out to contribute to the prepotential.

5. An $\mathcal{N} = 2$ example: the $\mathbf{Z}_3$ orientifold

Let us now consider the quiver gauge theory obtained by placing an orientifold O3-plane at a $\mathbf{C} \times \mathbf{C}^2/\mathbf{Z}_3$ orbifold singularity. In what follows we will use $\mathcal{N} = 1$ superspace notation. We first briefly repeat the steps that led to the constructions of such a quiver

[6]The gauge invariant quantity above can be rewritten as the Pfaffian of a suitably defined mesonic matrix.

theory in the seminal paper [39]. Define $\xi = e^{2\pi i/3}$ and let the generator of the orbifold group act on the first two complex coordinates as

$$g : \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \to \begin{pmatrix} \xi & 0 \\ 0 & \xi^{-1} \end{pmatrix} \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} , \tag{5.1}$$

while leaving the third one invariant. This preserves $\mathcal{N} = 2$ SUSY. The action of the generator g on the Chan-Paton factors is given by the matrix

$$\gamma(g) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & \xi^2 \end{pmatrix} . \tag{5.2}$$

The $\mathcal{N} = 2$ theory obtained this way, summarized in Fig. 5, is a three node quiver gauge theory with gauge groups $\mathrm{SU}(N_1) \times \mathrm{SU}(N_2) \times \mathrm{SU}(N_3)$, supplemented by a cubic superpotential which is nothing but the orbifold projection of the $\mathcal{N} = 4$ superpotential (its precise form is not relevant for the present purposes).

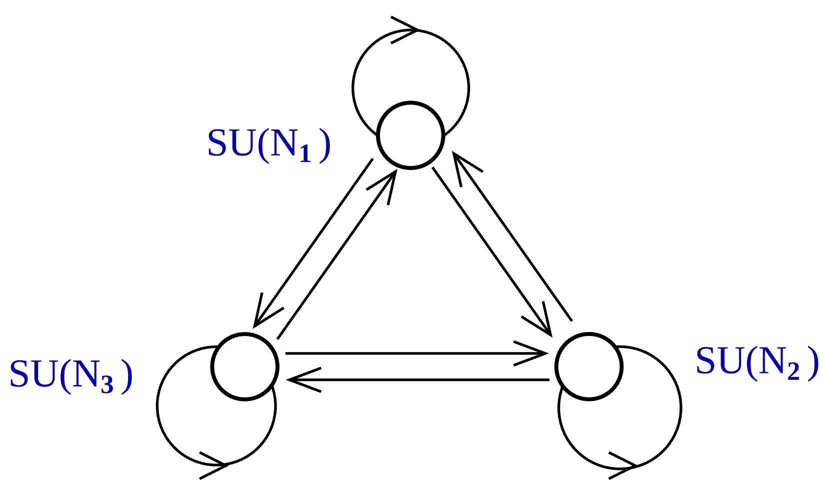

Figure 5: The $\mathbf{Z}_3$ (un-orientifolded) theory. The lines with both ends on a single node represent adjoint chiral multiplets which, together with the vector multiplets at each node constitute the $\mathcal{N} = 2$ vector multiplets. Similarly, lines between nodes represent chiral multiplets which pair up into hyper-multiplets, in $\mathcal{N} = 2$ language.

As for the action of Ω on the Chan-Paton factors, we choose again to perform the symplectic projection on the D3-branes. To do so, we must take N_1 to be even and $N_2 = N_3$, so that we can write

$$\gamma_-(\Omega) = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} , \tag{5.3}$$

where ϵ is a $N_1 \times N_1$ antisymmetric matrix obeying $\epsilon^2 = -1$ and the 1's denote $N_2 \times N_2$ identity matrices. The matrices $\gamma(g)$ and $\gamma_-(\Omega)$ satisfy the usual consistency condition [38, 39] as in (4.1).

The field content on the fractional D3-branes at the singularity will be given by implementing the conditions

$$
\begin{aligned}
A_\mu &= \gamma(g) A_\mu \gamma(g)^{-1} \quad , \quad \Phi^i = \xi^{-i} \gamma(g) \Phi^i \gamma(g)^{-1} \ , \\
A_\mu &= -\gamma_-(\Omega) A_\mu^T \gamma_-(\Omega)^{-1} \quad , \quad \Phi^i = -\gamma_-(\Omega) \Phi^{iT} \gamma_-(\Omega)^{-1} \ .
\end{aligned} \tag{5.4}
$$

The orbifold part of these conditions forces A_μ and Φ^3 to be 3×3 block diagonal matrices, *e.g.* $A_\mu = \mathrm{diag}\,(A_\mu^1, A_\mu^2, A_\mu^3)$, while the orientifold imposes that $A_\mu^1 = \epsilon A_\mu^{1T} \epsilon$ and $A_\mu^2 = -A_\mu^{3T}$. The resulting gauge theory is thus a $\mathrm{USp}(N_1) \times \mathrm{SU}(N_2)$ theory. It is convenient, however, to still denote A_μ^2 and A_μ^3 diagramatically as belonging to different nodes with the understanding that these should be identified in the above sense.

The projection on the chiral fields can be done similarly and we obtain, denoting by $\Phi_{\ell m}$ the non-zero entries of the fields Φ^1 and Φ^2 (only one can be non-zero for each pair ℓm)

$$
\Phi_{12} = -\epsilon \Phi_{31}^T, \quad \Phi_{13} = +\epsilon \Phi_{21}^T, \quad \Phi_{23} = \Phi_{23}^T, \quad \Phi_{32} = \Phi_{32}^T \ . \tag{5.5}
$$

The field content is summarized in Table 2.

	$\mathrm{USp}(N_1)$	$\mathrm{SU}(N_2)$
Φ_{12}	$\square$	$\overline{\square}$
Φ_{21}	$\square$	$\square$
Φ_{13}	$\square$	$\square$
Φ_{31}	$\square$	$\overline{\square}$
Φ_{23}	$\cdot$	$\square\!\square$
Φ_{32}	$\cdot$	$\overline{\square\!\square}$

Table 2: Chiral fields making up the quiver gauge theory.

The theory we want to focus on in the following has rank assignment $(N_1, N_2) = (0, N)$. This yields an $\mathcal{N} = 2$ $\mathrm{SU}(N)$ gauge theory with an hyper-multiplet in the symmetric/(conjugate)symmetric representation. We denote the $\mathcal{N} = 2$ vector multiplet by $\mathcal{A}$ whose field content in the block 3×3 notation is thus

$$
\hat{\mathcal{A}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathcal{A} & 0 \\ 0 & 0 & -\mathcal{A}^T \end{pmatrix} \ . \tag{5.6}
$$

In what follows we will be interested in studying corrections to the prepotential $\mathcal{F}$ coming from exotic instantons associated to the first node (the one that is not populated by D3-branes). Let us then analyze the structure of the stringy instanton sector of the present model, first.

5.1 Instanton sector

The most generic situation is to have a configuration with instanton numbers (k_1, k_2) (later we will be mainly concerned with a configuration with instanton numbers $(1,0)$).

Let us start analyzing the zero-modes content in neutral sector. The story is pretty similar to the one discussed in the previous section. The vertex operator for a_μ is proportional to $\partial_\sigma x^\mu$ and so it remains invariant under Ω. The action on the Chan-Paton factors of these D-instantons must now be represented by a symmetric matrix which we take to be

$$\gamma_+(\Omega) = \begin{pmatrix} 1' & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{5.7}$$

where $1'$ is a $k_1 \times k_1$ unit matrix and the 1's are $k_2 \times k_2$ unit matrices.

Because of the different orientifold projection, the matrices of bosonic zero-modes behave slightly differently. The matrices a_μ, χ^8 and χ^9 will still be 3×3 block diagonal, *e.g.* $a_\mu = \text{diag}\,(a^1_\mu, a^2_\mu, a^3_\mu)$, but now $a^1_\mu = a^{1T}_\mu$ and $a^2_\mu = a^{3T}_\mu$ whereas the same relations for χ^8 and χ^9 will have an additional minus sign. The remaining fields $\chi^{4\ldots7}$ are off diagonal and we shall not consider them further since we will consider only the case of one type of instanton. By considering a configuration with $k_1 = 1$ and $k_2 = 0$, we can project out all bosonic zero-modes except for the four components a^1_μ that we denote by x_μ.

Let us now consider the orientifold action on the fermionic neutral zero-modes $M^{\alpha A}$ and $\lambda_{\dot\alpha A}$. The orbifold part of the group acts on the internal spinor indices as a rotation

$$R(g) = e^{\frac{\pi}{3}\Gamma^{45}} e^{-\frac{\pi}{3}\Gamma^{67}} \ , \tag{5.8}$$

while the orientifold acts through the matrix $R(\Omega)$ given in (4.5). The orbifold and orientifold projections thus require

$$\begin{aligned} M^{\alpha A} &= R(g)^A{}_B \gamma(g) M^{\alpha B} \gamma(g)^{-1} \quad , \quad \lambda_{\dot\alpha A} = \gamma(g) \lambda_{\dot\alpha B} \gamma(g)^{-1} R(g)^B{}_A \ , \\ M^{\alpha A} &= R(\Omega)^A{}_B \gamma_+(\Omega) (M^{\alpha B})^T \gamma_+(\Omega)^{-1} \quad , \quad \lambda_{\dot\alpha A} = \gamma_+(\Omega) (\lambda_{\dot\alpha B})^T \gamma_+(\Omega)^{-1} R(\Omega)^B{}_A \ . \end{aligned} \tag{5.9}$$

Using the explicit expressions for the various matrices, we see that, for the simple case where $k_1 = 1$ and $k_2 = 0$, all λ's are projected out and only *four* chiral M zero-modes remain: $M^{\alpha---}$ and $M^{\alpha++-}$ to be identified with the $\mathcal{N} = 2$ chiral superspace

coordinates θ^1_α and θ^2_α. Hence, also in this case the orientifold projection has cured the problem encountered in section 3 (albeit in a $\mathcal{N} = 2$ context now) and we can rest assured that the integration over the charged modes will yield a contribution to the prepotential.

Let us now move to the charged zero-modes sector. Just as in the previous model, there are no bosonic modes since the D-instanton and the D3-branes sit at different nodes while the bosonic modes are necessarily diagonal. Most of the fermionic zero-modes μ^A and $\bar{\mu}^A$ are projected out by the orbifold condition which is formally the same as in (4.8), while the orientifold condition relates the fields in the conjugate sectors, giving $\bar{\mu}$ as a linear combination of the μ's according to

$$\bar{\mu}^A = R(\Omega)^A{}_B \gamma_+(\Omega)(\mu^B)^T \gamma_-(\Omega)^{-1} \ . \tag{5.10}$$

To summarize, the only charged modes surviving the projection can be expressed, in block 3×3 notation as

$$\begin{aligned} \mu^1 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mu & 0 & 0 \end{pmatrix} \quad , \quad \bar{\mu}^1 = \begin{pmatrix} 0 & \mu^T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \\ \mu^2 &= \begin{pmatrix} 0 & 0 & 0 \\ \mu' & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad , \quad \bar{\mu}^2 = \begin{pmatrix} 0 & 0 & -\mu'^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{5.11}$$

where the entries are to be thought of as column/row vectors in the fundamental/anti-fundamental of $\mathrm{SU}(N)$ depending on their position.

As anticipated, the configuration we want to consider is a $(0, N)$ fractional D3-branes system together with an exotic $(1, 0)$ instanton. The quiver structure, including the relevant moduli, is depicted in Fig. 6. It is now easy to see that inserting the expressions (5.6) and (5.11) into Eqs. (2.1), (2.3) and (2.7) we finally obtain

$$Z = \int dx^4 d\theta^4 \, \mathcal{F} \quad \text{with} \quad \mathcal{F} = \mathcal{C} \int d\mu d\mu' \ e^{i\mu^T \mathcal{A} \mu'} \propto \det \mathcal{A} \ . \tag{5.12}$$

It would be interesting to study the potential implications of this result in the gauge theory. There are many other simple models that could be analyzed along these lines.

6. Conclusions

In this paper we have presented some simple examples of what seem to be rather generic phenomena in the context of string instanton physics. We paid particular attention to

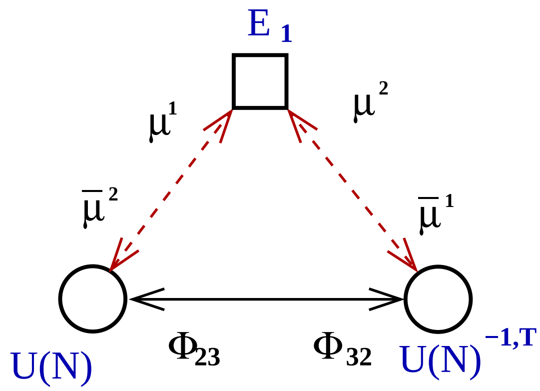

Figure 6: The extended $\mathbf{Z}_3$ orientifold theory with $(0, N)$ fractional D3-branes and $(1, 0)$ instanton number. The upper node (which would represent the $\mathrm{USp}(N_1)$ gauge group and disappears when we set $N_1 = 0$ as in the case under consideration) is where the instanton sits. The lower nodes denote only one gauge group. The charged fermionic zero-modes follow Eq. (5.11). For simplicity we have not drawn the lines denoting the adjoint.

the study of the fermionic zero-modes and their effects on the holomorphic quantities of the theory. We have seen both examples where the instanton contributions vanish due to the presence of extra zero-modes and where they do not. In the second case, as explicitly shown in a $\mathcal{N} = 1$ example, exotic instantons can have a stabilizing effect on the theory.

Although we have only considered some simple examples, we would like to stress that these results are quite generic and can be carried over to all orbifold gauge theories. A future direction would be to try to be more systematic and analyze the various possibilities encountered in more complex $\mathcal{N} = 2$ and $\mathcal{N} = 1$ models. In a similar spirit, one should analyze the multi-instanton contributions as well, since the total correction to the holomorphic quantities will be the sum of all such terms. The study of the zero-modes is expected to be even more relevant in this case as it will probably make many contributions vanish. With an eye to string phenomenology, one should also incorporate these models into globally consistent compactifications and study the effects of these terms there.

Lastly, it would be interesting to study the dynamical implications of some of the terms generated. We briefly touched upon this at the end of section 4 when we mentioned the stabilizing effect of the exotic instanton on the $\mathrm{USp}(N)$ theory. Although from the strict field theory point of view these terms are thought of as ordinary polyno-

mial terms in the holomorphic quantities,[7] they are "special" when seen from the point of view of string theory and they might therefore induce a particular type of dynamics.

Acknowledgements

We would like to thank many people for discussions and email exchanges at various stages of this work that helped us sharpen the focus of the presentation: M. Bianchi, M. Billò, P. Di Vecchia, S. Franco, M. Frau, F. Fucito, S. Kachru, R. Marotta, L. Martucci, F. Morales, B. E. W. Nilsson, D. Persson, I. Pesando, D. Robles-Llana, R. Russo, A. Tanzini, A. Tomasiello, A. Uranga, T. Weigand and N. Wyllard.

R.A., M.B. and A.L. are partially supported by the European Commission FP6 Programme MRTN-CT-2004-005104, in which R.A is associated to V.U. Brussel, M.B. to University of Padova and A.L. to University of Torino. R.A. is a Research Associate of the Fonds National de la Recherche Scientifique (Belgium). The research of R.A. is also supported by IISN - Belgium (convention 4.4505.86) and by the "Interuniversity Attraction Poles Programme –Belgian Science Policy". M.B. is also supported by Italian MIUR under contract PRIN-2005023102 and by a MIUR fellowship within the program "Rientro dei Cervelli". The research of G.F. is supported by the Swedish Research Council (Vetenskapsrådet) contracts 622-2003-1124 and 621-2002-3884. A.L. thanks the Galileo Galilei Institute for the hospitality and support during the completion of this work.

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